Optimal Transport and Geometric Analysis

ABSTRACTS

Monday, 1st April

Luigi Ambrosio (*Scuola Normale Superiore*)
Title: **Semigroups and Geometric Measure Theory**
Abstract: I will illustrate with a few examples how semigroup tools can have a crucial role in the proof of extensions of classical results in Geometric Measure Theory and Real Analysis.

Tobias Colding (*MIT*)
Title: **A new approach to higher codimension mean curvature flow**
Abstract: We will discuss very recent joint work with Bill Minicozzi about a new approach to higher codimension mean curvature flow. This is a subject that has been notoriously difficult and where much less is known than for hypersurfaces. Some of the inspiration for this new approach comes from function theory on manifolds with Ricci curvature bounds.

Felix Schulze (*UCL London*)
Title: **Uniqueness of asymptotically conical tangent flows**
Abstract: Singularities of the mean curvature flow of an embedded surface in R^3 are expected to be modelled on self-shrinkers that are compact, cylindrical, or asymptotically conical. In order to understand the flow before and after the singular time, it is crucial to know the uniqueness of tangent flows at the singularity. In all dimensions, assuming the singularity is multiplicity one, uniqueness in the compact case has been established by Schulze, and in the cylindrical case by Colding–Minicozzi. We show here the uniqueness of multiplicity-one asymptotically conical tangent flows for mean curvature flow of hypersurfaces. In particular, this implies that when a mean curvature flow has a multiplicity-one conical singularity model, the evolving surface at the singular time has an (isolated) regular conical singularity at the singular point. This should lead to a complete understanding of how to “flow through” such a singularity. This is joint work with Otis Chodosh.

Giuseppe Savaré (*Università di Pavia*)
Title: **Density of algebras of functions in metric Sobolev spaces**
Abstract: We will discuss a possible construction of metric Sobolev spaces in extended metric-topological structures, showing sufficient conditions to guarantee the density of suitable algebras of bounded Lipschitz functions. As a byproduct we will also obtain a new proof of the density of Lipschitz functions which relies on a duality argument and does not involve the gradient flow of the Cheeger energy.

Daniele Semola (*Scuola Normale Superiore*)
Title: **Constancy of the dimension for RCD(K,N) spaces via regularity of Lagrangian flows**
Abstract: After a brief review about the structure theory of RCD(K,N) spaces I will show how some regularity results for Lagrangian flows of Sobolev vector fields can be used to extend to this framework (with a new proof) the constancy of the dimension theorem proved by Colding-Naber for Ricci-limits. This is based on a joint work with Elia Bruè.

Raquel Perales (*Unam Oaxaca*)
Title: **Maximal Volume Entropy Rigidity for RCD(-(N-1),N)**
(Joint work with Connell, Dai, Nunez-Zimbron, Suarez-Serrato, Wei)
Abstract: For n-dimensional Riemannian manifolds M with Ricci curvature bounded below by -(n-1), the volume entropy is bounded above by n-1. If M is compact, it is known that the equality holds if and only if M is hyperbolic. The same lower bound on the volume entropy is easily obtained for RCD(-(N-1),N) spaces. Thus, in this talk, I will discuss how to obtain the rigidity result.

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Tuesday, 2nd April (morning)

Robert McCann (*University of Toronto*)
Title: **Displacement convexity of Boltzmann's entropy characterizes positive energy in general relativity**
Abstract: Einstein's theory of gravity is based on assuming that the fluxes of an energy and momentum in a physical system are proportional to a certain variant of the Ricci curvature tensor on a smooth 3+1 dimensional spacetime. The fact that gravity is attractive rather than repulsive is encoded in the positivity properties which this tensor is assumed to satisfy. Hawking and Penrose (1971) used this strong positivity of energy to give conditions under which smooth spacetimes must develop singularities.
By lifting fractional powers of the Lorentz distance between events in a globally hyperbolic spacetime to probability measures on these events, we show that the strong energy condition of Hawking and Penrose is equivalent to convexity of the Boltzmann-Shannon entropy along the resulting geodesics of probability measures. This new characterisation of the strong energy condition on globally hyperbolic manifolds also makes sense in (non-smooth) metric measure settings, where it has the potential to provide a framework for developing a theory of gravity which admits certain singularities and can be continued beyond them. It provides a Lorentzian analog of Lott, Villani and Sturm's metric-measure theory of lower Ricci bounds, and hints at new connections linking gravity to the second law of thermodynamics.

Wenshuei Jiang (*Zhejiang University*)
Title: **Rectifiability of Singular Sets in Lower Ricci Curvature**
Abstract: In this talk, we will consider the Gromov-Hausdorff limit space (X,d) of a sequence of n-manifolds with lower Ricci curvature bound and noncollapsed volume. The limit space has a singular-regular decomposition X=R\cup S and the singular set S has dimension <=n-2 proved by Cheeger-Colding. In this talk we will study the structure of the singular set S and show that the singular set is (n-2)-rectifiable. We will also discuss the quantitative estimate of the singular set. The proofs are based on some new estimates on neck regions and a decomposition theorem which covers a general ball by neck regions and good balls. This is a joint work with Jeff Cheeger and Aaron Naber.

Stefan Suhr (*University of Bochum*)
Title: **Lorentzian optimal transportation and the Einstein equation**
Abstract: The Ricci curvature is the basic ingredient in the Einstein equations of general relativity. In recent years the interpretation of Ricci curvature in Riemannian geometry has changed fundamentally via its characterization in terms of convexity properties of e.g. the Shannon-Bolzmann entropy of optimal transportation. In my talk I will explain the recent development of optimal transportation in Lorentzian geometry and an analogous characterization of Ricci curvature and the Einstein equation in Lorentzian geometry.

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Tuesday, 2nd April (afternoon)

Peter Topping (*University of Warwick*)
Title: **Ricci flows with rough initial data**
Abstract: TBA.

Ilaria Mondello (*Université de Paris Est Créteil*)
Title: **New geometric examples of RCD spaces**
Abstract: In the recent years, many striking results have been proven in the setting of metric measure spaces satisfying an RCD(K,N) curvature-dimension condition, which share a multitude of geometric and analytic properties with Riemannian manifolds carrying a lower Ricci bound and their Gromov-Hausdorff limits. In this talk we are interested in spaces that are in-between the smooth world of Riemannian manifolds and the very general setting of RCD spaces, that is singular manifolds, not necessarily arising as GH-limits. We will consider stratified spaces, which generalize manifolds with isolated conical singularities, and show a geometric criterion for them to satisfy the RCD(K,N) condition or being Alexandrov spaces. This is a joint work with J. Bertrand (Université Paul Sabatier, Toulouse), C. Ketterer (University of Toronto) and T. Richard (Université Paris Est Créteil).

Eva Kopfer (*University of Bonn*)
Title: **Variational limits of discrete and constrained optimal transport**
Abstract: First, we consider dynamical transport metrics for probability measures on discrete sets. These metrics appear as a natural counterpart of the Wasserstein distance. Second, we consider the problem of dynamic optimal transport with density constraint. In both settings we compute variational limits in terms of Gamma-convergence. The limit object we obtain in both cases is a Wasserstein-like distance with homogenized costs.

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Wednesday, 3rd April

Gerard Besson (*Institut Fourier-Grenoble*)
Title: **Entropy of closed manifolds and of their fundamental group**
Abstract: We will discuss the following question: is there any relation between the entropy of a closed manifold and the algebraic entropy of its fundamental group? The answer is (clearly) negative in general. However, for the case of Gromov-hyperbolic spaces we describe some instances in which it is true. There is an inequality between the two invariants.

Martin Kell (*Tubingen University*)
Title: **Existence and non-existence of dual solutions for Lorentzian cost functionals**
Abstract: (joint with S. Suhr) The existence of a dual solution is an important ingredient to unravel the power of optimal transport. Dual solutions turn out to be essential in classical existence proofs of optimal transport maps and are used to obtain volume distortion inequalities whose lower bounds are determined by lower bounds on the Ricci curvature. Only recently, new existence proof of optimal transport maps were found that do not depend anymore on the existence of dual solutions, both in the classical Lp-Monge problems as well as the Lorentzian equivalents. In this talk I will show that in the Lorentzian setting optimal couplings that transport positive mass along the light cone cannot admit dual solutions. Nevertheless, the classical construction of dual solutions gives a hint to obtain conditions for which dual solutions exist.

Shouhei Honda (*Tohoku University*)
Title: **Embedding of RCD(K, N) space in L^2 via eigenfunctions**
Abstract: Berard-Besson-Gallot proved that any closed Riemannian manifold can be embedded in L^2 via the heat kernel and that the original Riemannian metric can be approximated by the pull-back metrics. In this talk we generalize this theorem to singular spaces, so-called RCD(K, N) metric measure spaces. Combining the Gromov-Hausdorff compactness of the moduli space of noncollapsed spaces with Reifenberg flatness, we prove a quantitative sharp convergence result for the pull-back metrics, which is new even for closed Riemannian manifolds. This is a joint work with L. Ambrosio, J. W. Portegies and D. Tewodrose.

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Thursday, 4th April

Alexander Lytchak (*University of Koln*)
Title: **RCD in 2D**
Abstract: In the talk, I would like to discuss the following result obtained jointly with Stephan Stadler. Any two-dimensional RCD space with the Hausdorff area as its measure is an Alexandrov space.

Nicola Gigli (*SISSA*)
Title: **Functional analysis and metric geometry**
Abstract: Aim of the talk is to present some aspects of the important role that functional analysis has in the context of metric geometry. I shall discuss both the case of synthetic description of lower Ricci curvature bounds, where this role is by now well understood, and some potential applications to the world of lower sectional curvature bounds, where it might potentially lead to the solution of long-standing open problems.

Fabio Cavalletti (*SISSA*)
Title: **Some properties of spaces verifying the Measure-Contraction property**
Abstract: We review some recent results concerning the family of metric measure spaces verifying the so called Measure-Contraction property. Most notably examples of spaces verifying this condition include Heisenberg group and other sub-Riemannian spaces. We will discuss a measure-theoretic splitting, Laplacian comparison (joint with A. Mondino) and a sharp isoperimetric inequality á la Levy-Gromov (joint with F. Santarcangelo).

Christian Ketterer (*Toronto University*)
Title: **Stability of graphical tori with almost non-negative Scalar curvature**
Abstract: The Scalar curvature torus rigidity theorem states that a Riemannian torus with non-negative Scalar curvature must be flat. The case for dimension less than 8 was established by Schoen and Yau using minimal surface techniques, the general case was established by Gromov and Lawson using the Lichnerowicz identity for spin manifolds. Gromov conjectured that a sequence of Riemannian tori with almost non-negative Scalar curvature and suitably normalized subconverges to a flat torus w.r.t. to a weak "Sobolev-type" topology on the class of Riemannian manifolds. In this talk, we confirm Gromov's conjecture for a special class of 3-dimensional graphical tori and the Sormani-Wenger intrinsic flat topology. This is a joint work with Raquel Perales and Armando Cabrera-Pacheco.

Fernando Galaz-Garcia (*KIT*)
Title: **Sufficiently collapsed three-dimensional Alexandrov spaces**
Abstract: In Riemannian geometry, collapse imposes strong geometric and topological restrictions on the spaces on which it occurs. In the case of Alexandrov spaces, which are metric generalizations of complete Riemannian manifolds with a uniform lower sectional curvature bound, collapse is fairly well understood in dimension three. In this talk, I will discuss the geometry and topology of sufficiently collapsed three-dimensional Alexandrov spaces: when the space is irreducible, it is modeled on one of the eight three-dimensional dimensional Thurston geometries, excluding the hyperbolic one. This extends a result of Shioya and Yamaguchi, originally formulated for Riemannian manifolds, to the Alexandrov setting. (Joint work with Luis Guijarro and Jesús Núñez-Zimbrón).

Matthias Erbar (*University of Bonn*)
Title: **Super Ricci flows for discrete spaces**
Abstract: I will present a discrete notion of super Ricci flow that applies to time dependent Markov chains or weighted graphs. This notion can be characterised equivalently in terms of a discrete time-dependent Bochner inequality, gradient estimates for the heat propagator on the evolving graph, contraction estimates in discrete transport distances, or dynamic convexity of the entropy. I will also discuss several examples. This is joint work with Eva Kopfer.

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Friday, 5th April

John Lott (*UC Berkeley*)
Title: **On Ricci-pinched 3-manifolds**
Abstract: There is a conjecture that a complete Riemannian 3-manifold with pointwise pinched nonnegative Ricci curvature must be flat or compact. I will show that this is true provided that the negative sectional curvature (if any) decays quadratically.

Elia Brué (*Scuola Normale Superiore*)
Title: **Rigidity of the 1-Bakry-Émery inequality and sets of finite perimeters in RCD spaces**
Abstract: It is well-know that, in spaces with nonnegative Ricci curvature, a function satisfying the equality in the Bakry-Emery contraction estimate with exponent p=2 is a splitting function. Unfortunately, this is not anymore the case when one considers the inequality with exponent p=1. However, in the setting of RCD spaces this weaker rigidity comes up naturally in the study of fine properties of sets with finite perimeter. In this talk I present a recent work in collaboration with L. Ambrosio and D. Semola in which we prove a splitting result in the critical case p=1 and we use it as a main tool to describe tangents of sets with finite perimeter.

Christina Sormani (*CUNY and Lehman College*)
Title: **Ricci Curvature and Intrinsic Flat Convergence**
Abstract: Recently Matveev-Portegies have proven that noncollapsing sequences of manifolds with uniform lower bounds on their Ricci curvature converge in the Gromov Hausdorff (GH) and Sormani-Wenger Intrinsic Flat (SWIF) senses to the same limit space. Perales has similar results when the manifolds have boundary assuming a variety of conditions on the boundary. I will review SWIF convergence and its properties, these theorems, and state some open problems. See https://sites.google.com/site/intrinsicflatconvergence/ for the relevent papers.
(Substitute presenter: Raquel Perales)